MATH 3150 – HOMEWORK 9

Problem 1 (p. 191, #4)**.** Let $f : A \subset \mathbb{R}^n \longrightarrow \mathbb{R}$ be continuous, $x, y \in A$ and $c : [0,1] \longrightarrow A \subset \mathbb{R}^n$ be a continuous curve joining x and y. Show that along this curve, f attains its maximum and minimum values (among all values along the curve).

Problem 2 (p. 193, #3)**.** Let $f : [0,1] \longrightarrow [0,1]$ be continuous. Prove that f has a fixed point (i.e. a point $x \in [0,1]$ such that $f(x) = x$).

Problem 3 (p. 196, #2)**.** Prove that $f(x) = 1/x$ is uniformly continuous on $[a, \infty)$ for any $a > 0$.

Problem 4 (p. 232, #12)**.** A map $f : A \subset \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is called *Lipschitz on A* if there is a constant $L \geq 0$ such that $\|f(x) - f(y)\| \leq L \|x - y\|$ for all $x, y \in A$.

(a) Show that a Lipschitz map is uniformly continuous.
(b) Find a bounded continuous function $f : \mathbb{R} \longrightarrow \mathbb{R}$ that is not uniformly continuous and hence is not Lipschitz.
(c) Is the sum of two Lipschitz functions again a Lipschitz function?
(d) Is the product of two Lipschitz functions again a Lipschitz function?
(e) Is the sum of two uniformly continuous functions uniformly continuous?
(f) Is the product of two uniformly continuous functions uniformly continuous?