Math 2420 – Problem Set 4, due Monday 4/9.

Problem 1. As mentioned in class, weak equivalence is not itself an equivalence relation, i.e. just because there exists a map $f : X \longrightarrow Y$ such that $f_* : \pi_n(X) \cong \pi_n(Y)$ for all n does not mean there is such a map $g : Y \longrightarrow X$. However, it does define an equivalence relation through the following construction. Say $X \simeq_w Y$ if there exists a finite sequence of spaces $X = X_1, X_2, \ldots, X_n = Y$ such that there are weak equivalences either $X_i \longrightarrow X_{i+1}$ or $X_{i+1} \longrightarrow X_i$ for each i. Show that this is an equivalence relation, and prove that $X \simeq_w Y$ if and only if X and Y have a common CW approximation.

Problem 2. Show that a map $f : X \longrightarrow Y$ between connected CW complexes factors as a composition $X \longrightarrow Z_n \longrightarrow Y$ where the first map induces isomorphisms on π_i for $i \leq n$ and the second map induces isomorphisms on π_i for $i \geq n + 1$.

Problem 3. Given an abelian group G, give a construction of the Eilenberg-MacLane space $K(G, n)$ as a CW complex.